Oscillations and Waves

IB 12

Oscillation: the movement of an object to-and-fro between 2 points, around an average position

Wave: a method of transferring energy without transferring matter

Examples of oscillations:

1. mass on spring (eg. bungee jumping)
2. pendulum (eg. swing)
3. object bobbing in water (eg. buoy, boat)
4. vibrating cantilever (eg. diving board)
5. earthquake
6. bouncing ball
7. musical instruments (eg. strings, percussion, brass, woodwinds, vocal chords)
8. heartbeat

Mean Position (Equilibrium Position) – position of object at rest

Displacement (x, meters) – distance in a particular direction of a particle from its mean position

Amplitude (A or x_0, meters) – maximum displacement from the mean position

Period (T, seconds) – time taken for one complete oscillation

Frequency (f, Hertz) – number of oscillations that take place per unit time

Phase Difference – difference in phase between the particles of two oscillating systems, measured in radians

Relationship between period and frequency: $f = \frac{1}{T}$

$f \equiv \text{cycles}/\text{sec} = 1/s = s^{-1} = \text{Hz}$

$T \equiv \text{sec}$

Angular Frequency -

Formula: $\boxed{\omega = 2\pi f}$

$\omega = f \frac{\text{cycles}}{\text{sec}} \cdot \frac{2\pi \text{ radians}}{\text{cycle}}$

Symbol: ω (omega)

Units: $\frac{\text{radians}}{\text{sec}}$

1. A pendulum completes 10 swings in 8.0 seconds.

a) Calculate its period.

$T = \frac{8.0\,s}{10 \text{ cycles}} = 0.80\ s$

b) Calculate its frequency.

$f = \frac{1}{T} = 1.25\ \text{Hz}$

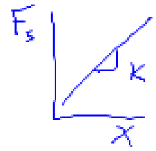
c) Calculate its angular frequency.

$\omega = 2\pi f = 7.8 \frac{\text{rad}}{\text{sec}} = 7.8\ s^{-1}$

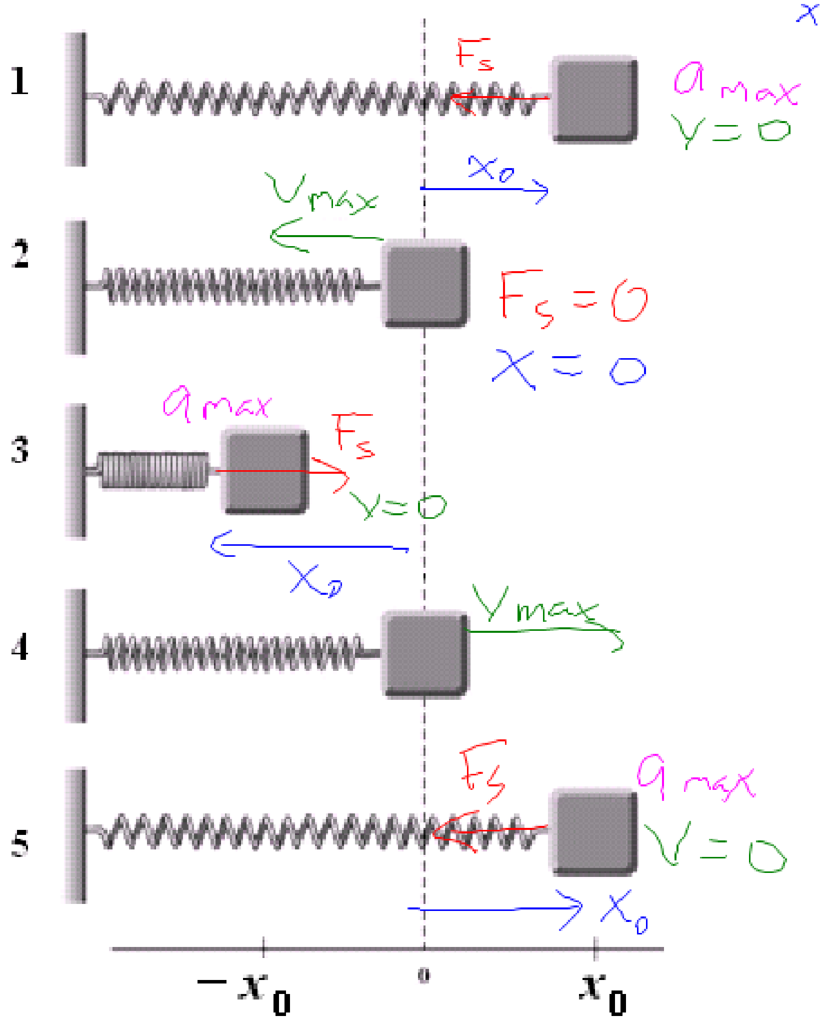
Example of an Oscillating System

A mass oscillates on a horizontal spring without friction as shown below. At each position, analyze its displacement, velocity and acceleration.

Force from the Spring: Hooke's Law $\vec{F_s} = -k\vec{x}$



the spring force is proportional to the displacement (from equil) and in the opposite direction



1. When is the velocity of the mass at its maximum value?

positions 2 & 4 when $x=0$ (at equilibrium)

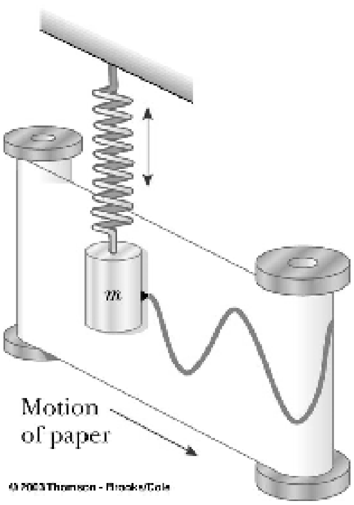
2. When is the acceleration of the mass at its maximum value?

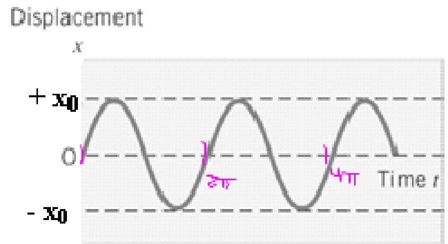
positions 1, 3, 5 $x = x_0$

because this is where F_s is maximum

The Displacement Function

A mass on a spring is allowed to oscillate up and down about its mean position without friction.
Two traces of the displacement (x) of the mass versus time (t) are shown.





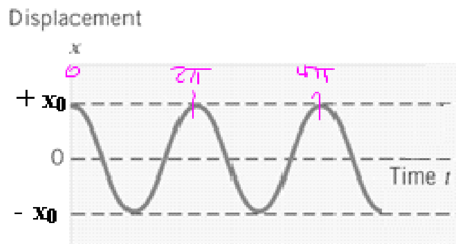
Initial condition: $t=0,\ x=0$

Function: $x = x_0 \sin\theta$

$\theta = \omega t$

angular frequency

$x = x_0 \sin(\omega t)$



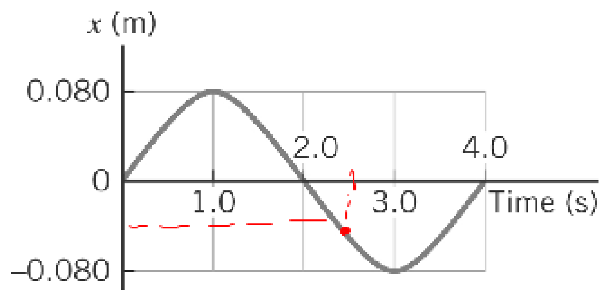
Initial condition: $t=0,\ x=x_0$

Function: $x = x_0 \cos\theta$

$x = x_0 \cos(\omega t)$

Analyzing the Displacement Function

1. Analyze the displacement function shown at right.



a) What is the amplitude?

$x_0 = 0.080\ m$

a) What is the period?

$T = 4.0\ s$

b) What is the frequency?

$f = \frac{1}{T} = \frac{1}{4.0\ s} = \underline{0.25}\ Hz$

c) What is the angular frequency?

$\omega = 2\pi f = 2\pi(0.25\ Hz)$

$= \frac{\pi}{2}\ s^{-1} = 1.57\ s^{-1}$

e) Write the displacement function.

$x = x_0 \sin(\omega t)$

$x = (0.080\ m)\sin\left(\frac{\pi}{2}t\right)$

2. What is the displacement of the mass when:

a) $t = 1.0$ s?

$x = (0.080)\sin\left(\frac{\pi}{2}(1.0s)\right)$

$= (0.080\ m)(1)$

$= 0.080\ m$

b) $t = 2.0$ s?

$x = (0.080\ m)\sin\left(\frac{\pi}{2}s^{-1}(2s)\right)$

$= (0.080\ m)(0)$

$= 0$

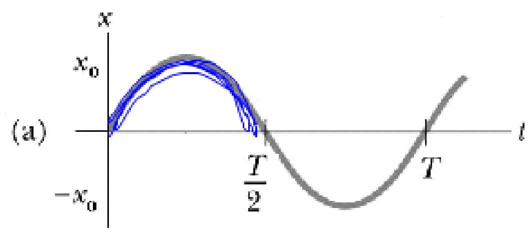
c) $t = 2.5$ s?

$x = 0.080\ m \sin\left(\frac{\pi}{2}s^{-1}(2.5s)\right)$

$= 0.0800\ m\ (-0.707)$

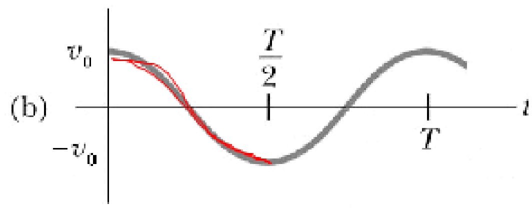
$= -0.057\ m$

Velocity and Acceleration for Simple Harmonic Motion



a) Displacement Function

$$X = X_0 \sin(\omega t)$$

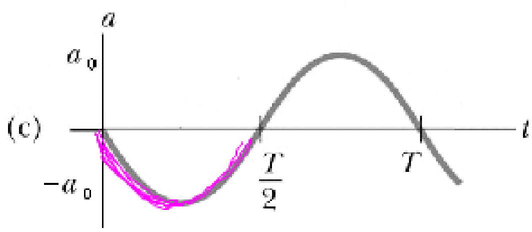


b) Velocity Function

$$V = V_0 \cos(\omega t)$$

where $V_0 = \omega X_0$ from UCM

$$\boxed{V = \omega X_0 \cos(\omega t)}$$



c) Acceleration Function

$$a = -a_0 \sin \omega t$$

where $a_0 = \omega^2 X_0$ from U.C.M.

$$\boxed{\begin{aligned} a &= -\omega^2 \underbrace{X_0 \sin \omega t}_{X} \\ a &= -\omega^2 X \end{aligned}}$$

Defining Equation for SHM:

acceleration is proportional to displacement (x) but in the opposite direction

$$a = -\omega^2 X$$

$\omega \equiv$ proportionality constant

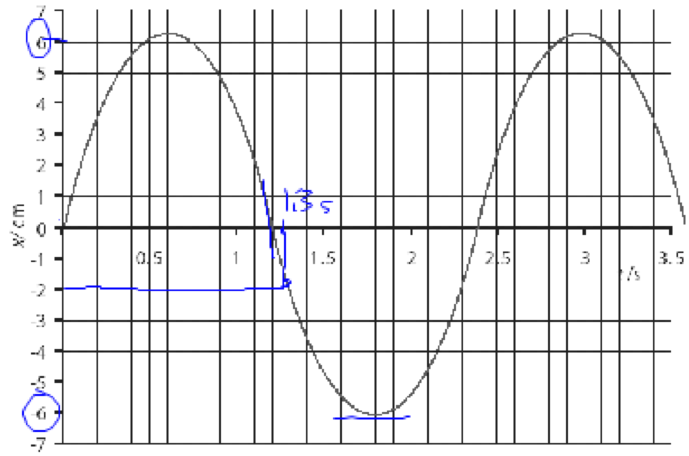
Negative Sign:

1) acceleration is in the opposite direction of displacement

2) accel. is directed back towards the equilibrium position

Simple Harmonic Motion (SHM) –

1. The graph shown at right shows the displacement of an object in SHM. Use the graph to find the:



a) period of oscillation

from the graph, $\boxed{T = 2.4\ s}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2.4\ s} = 2.6\ \frac{rad}{s}$

b) amplitude of oscillation

from the graph, $\boxed{X_0 \approx 6.0\ cm}$

c) displacement function

$X = X_0 \sin(\omega t)$

$X = (6.0\ cm)\sin(2.6\ s^{-1}\ t)$

d) maximum velocity

$V_0 = \omega X_0$

$= (2.6\ s^{-1})(6.0\ cm)$

$V_0 = 15.6\ cm/s$

e) velocity at 1.3 seconds

$V = V_0 \cos(\omega t)$

$V = (15.6\ cm/s)\cos(2.6\ s^{-1}\ t)$

$= (15.6\ cm/s)\cos(2.6\ s^{-1}(1.3\ s))$

$= -15.2\ cm/s$

f) maximum acceleration

$a_0 = \omega^2 X_0$

$= (2.6\ s^{-1})^2(6.0\ cm)$

$= 40.6\ cm/s^2$

g) acceleration at 1.3 seconds

$a = -a_0 \sin(\omega t)$

$= -(\omega^2 X_0)\sin(\omega t)$

$a = -(40.6\ cm/s^2)\sin(2.6\ s^{-1}(1.3\ s))$

$= +9.6\ cm/s^2$

Alternate Velocity Function

avg. velocity $\bar{V} = \frac{\Delta X}{\Delta t}$

instantaneous velocity = slope of tangent line

2. Use the alternate form of the velocity function to find the velocity of the object at 1.3 s.

$V = \frac{\Delta X}{\Delta t}$

$= \left(\frac{(-7\ cm - (+7\ cm))}{(1.6\ s - 0.7\ s)}\right)$

$= -15.5\ cm/s$

Example of SHM – Mass on a Horizontal Spring

A mass m oscillates horizontally on a spring without friction, as shown. Is this SHM?

Newton's 2nd $F_{net} = ma$

$F_s = ma$

From Hooke's Law $F_s = -kx$

$-kx = ma$

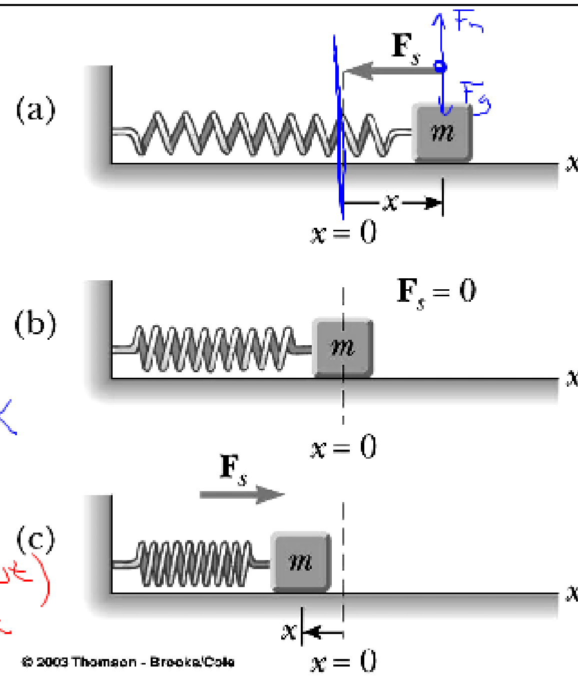
$$\boxed{a = -\left(\frac{k}{m}\right)x}$$

∴ Yes, this is SHM because $a \propto -x$

Note: For SHM $\boxed{a = -\omega^2 x}$

$= -\omega^2 (x_o \sin \omega t)$

$= -\omega^2 x_o \sin \omega t$



Angular frequency, period, and frequency for a mass on a spring

since $a = -\frac{k}{m}x$

and $a = -\omega^2 x$

then $\omega^2 = \frac{k}{m}$

or $\omega = \sqrt{\frac{k}{m}}$

$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega}$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

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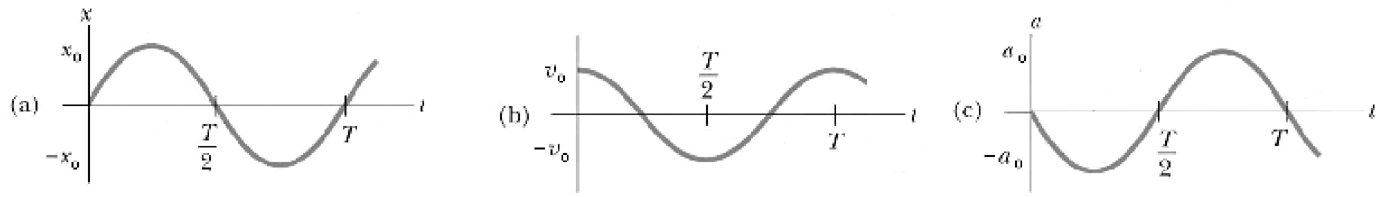
1. A 2.00 kg mass oscillates back and forth 0.500m from its rest position on a horizontal spring whose constant is 40.0 N/m.

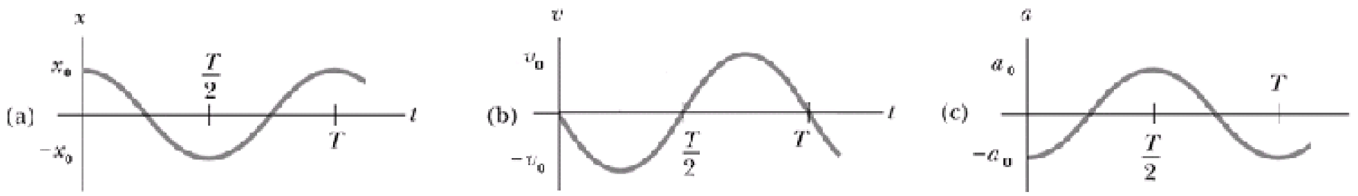
   a) Calculate the angular frequency, period and frequency of this system.

   b) Write the displacement, velocity and acceleration functions for this system.

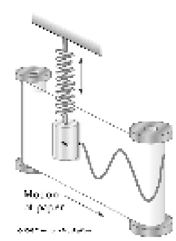
1. Write the equations of motion for the graphs shown below.



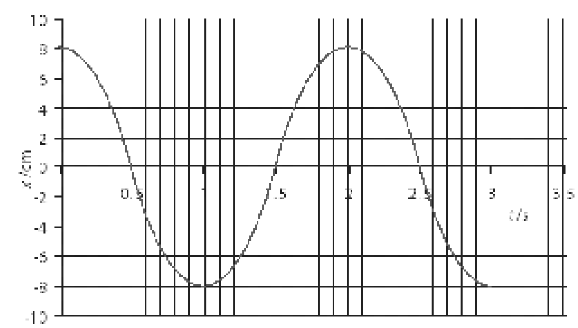
2. Write the equations of motion for the graphs shown below.



3. What is the difference between the motions described by the two sets of equations?



4. a) Write the equations of motion for the system whose displacement is shown on the graph at right.



b) State two times when the:

i) speed is maximum

ii) magnitude of the acceleration is maximum.